BNAM2012
Joint Baltic-Nordic Acoustics Meeting
June 18th - 20th 2012 Odense, Denmark

Prediction of Vibration Transmission within Periodic Bar Structures: Analytical Vs Numerical Approach

Parthkumar G. Domadiya and Lars V. Andersen
Aalborg University, Department of Civil Engineering,Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark
pdg@civil.aau.dk

Sergey V. Sorokin
Aalborg University, Department of Mechanical and Manufacturing Engineering, Pontoppidanstræde 103, DK-9220, Denmark

The present analysis focuses on vibration transmission within semi-infinite bar structure. The bar is consisting of two different materials in a periodic manner. A periodic bar model is generated using two various methods: The Finite Element method (FEM) and a Floquet theory approach. A parameter study is carried out regarding the influence of the number of periods at various frequencies within a semi-infinite bar, stop bands are illustrated at certain periodic intervals within the structure. The computations are carried out in frequency domain in the range below 500 Hz. Results from both of the above methods are compared and analyzed

1 Introduction

Fulfilling the acoustic requirements to automobiles, aircrafts and building structures is important. Numerical based models for vibro-acoustic problems leads to large number of linear equations. This is a significant drawback for mid and high frequency analysis. In this regard, statistical energy analysis (SEA) has shown better accuracy on prediction of noise transmission [1]. However, SEA has limited validity for lightweight structures such as wooden floors with joists spanning in one direction or double-plate panel walls with vertical ribs [2, 3]. The stiffeners lead to periodic nature of a structure, which provides a nonhomogeneous modal density due to formation of stop bands, and vibrations are not diffuse. Hence, other numerical methods of analysis must be employed.

As an alternative to SEA, the finite-element method (FEM) can be used [4] to describe flanking transmission in dwellings. Numerical simulations can reduce the cost of experiments and may also improve the design of sound insulation. However, the FEM has limitations when it comes to the high-frequency range. Small elements must be employed in order to obtain an adequate discretization of the waves propagating in the structure and the acoustic medium. This results in a huge number of degrees of freedom, leading to long computation times.

Currently, there is also an increasing interest in periodic structures for better sound insulation. By a theoretical study, Takahashi [5] found that the spacing between ribs and the stiffness of the connectors as well as the use of thick rigid materials all have a significant importance regarding the minimization of sound radiation from periodically connected infinite double-plate structures. Floquet theory has been used by several researchers [6, 7, 8 and 9] to evaluate vibration transmission behaviour of periodic structures. It has shown great performance in prediction on wave propagation in periodic structures. Floquet theory and FEM are employed in the current research and results are evaluated and compared.

The paper focuses on vibration transmission within a periodic bar structure which is made from two materials. Parametric analysis is done considering gradually increasing periodicity within the beam structure. The cross-sectional area is the same along the entire bar, since only longitudinal waves along the bar are considered; the cross-sectional area has no impact on the problem i.e. the wavelengths and phase velocities are not influenced. The analyses concern the dynamic response of beam structure to wave inputs and investigation of stop bands due to the periodic nature of the structure in the frequency range 0 to 500 Hz.

2 Problem Overview

Lightweight structures are usually constructed as panels with plates on stud or joist frames. In the present case, a semi-infinite periodic bar structure has been considered. The structure consists of two various materials connected with each other in a periodic manner. The aim of the study is to investigate the vibration transmission throughout bar with varying periodicity of the structure to see the effect of increased periodicity in transmission of vibrations. Analyses are carried out in frequency domain using finite element analysis and Floquet theory. Results from the methods are compared and investigated thoroughly.

2.1 Model Description and Materials

The one dimensional structure consists of a number of identical cells, each consisting of two materials. The material properties correspond to Plexiglas and Steel, and the Plexiglas and Steel part lengths are 60 mm and 30 mm respectively. The material properties are:

- Material 1 (Plexiglas): Young's modulus, E_1 = 3.4 GPa; mass density, ρ_1 = 1190 kg/m^3
- Material 2 (Steel): Young's modulus, E_2 = 200 GPa, mass density, ρ_2 =7850 kg/m^3

2.2 Finite Element Model

A one-dimensional semi-infinite bar structure is constructed from Plexiglas and Steel segments using a simple finite-element (FE) model. An FE code is generated based on the governing equations of motion for linear elastic wave propagation in a bar and finite elements with two nodes and linear interpolation of the displacement field are considered. Each node has a single degree of freedom, since only the longitudinal wave propagation is analysed. Figure 1 shows the complete formation of the semi-infinite beam structure. Firstly, mass and stiffness matrices for Plexiglas and steel elements are constructed and coupled (i.e. assembled) into one single cell. Secondly, the system matrices are formulated by assembling a number of cell matrices. To study the effects of increased periodicity in the bar, either 2, 4, 6, 8 or 10 cells are coupled in this manner. Finally, the boundary conditions are introduced, and the problem is solved in the frequency domain at the discrete frequencies 1, 2, …, 500 Hz. At one end of the periodic structure, a nodal force with unit magnitude is applied. At the other end, an impedance condition is introduced to mimic the behaviour of a semi-infinite bar consisting of material 1, i.e. Plexiglas.

The number of elements per wavelength is considered as 10, which satisfies that linear elements should be about one-tenth of the wavelength or smaller to avoid significant inaccuracies. The goal is to predict the characteristics of vibration transmission along the bar for harmonic motion at each individual frequency. The results are compared with the propagation characteristics predicted by Floquet theory for an infinite periodic structure.

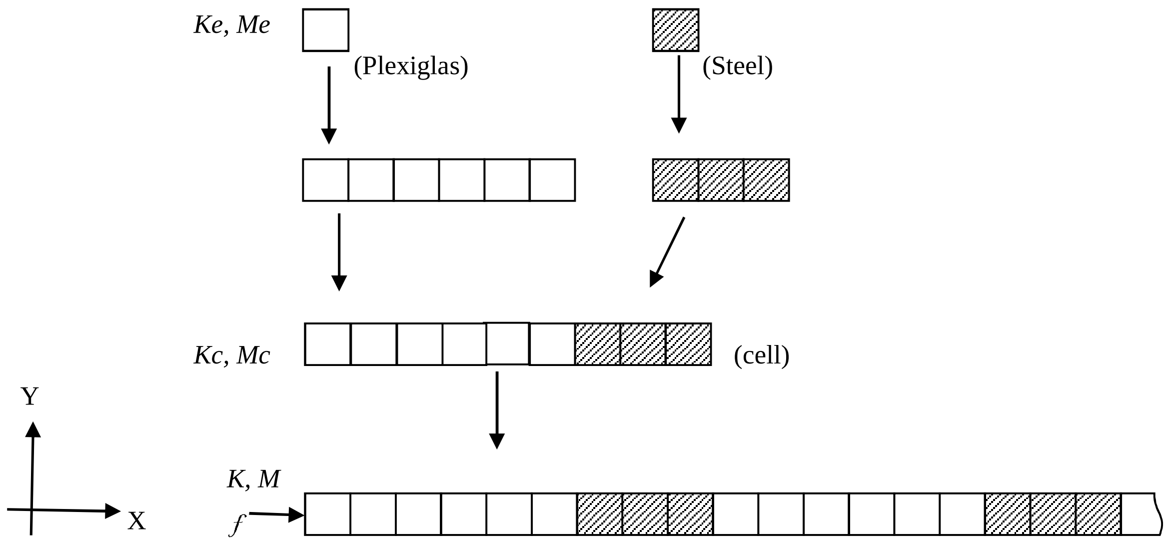

Figure 1: Formation of semi-infinite beam structure using FEM approach (*Ke*, *Me* = Stiffness and Mass element matrices, *Kc, Mc* = Stiffness and Mass cell matrices, *K, M* = Stiffness and Mass system matrices

2.3 Floquet Approach

Prediction of wave motion within periodic structures can be done using Floquet theory which eliminates the need to formulate the transfer matrices for the periodic elements. In the present case, the infinite bar is constructed by a periodic repetition of two different segments composed of Plexiglas and Steel, respectively. Each of the segments is specified by the following parameters: the segment length L_j, Young's Modulus E_j, the mass density ρ_j, the cross-sectional area A_j, and the displacements $u_j(x)$, j = 1, 2. Thus, the mass per unit length along the bar becomes $\mu_j=\rho_j A_j$, and the inverse of the phase velocities is given as $\alpha_j = \sqrt{\frac{\rho_j}{E_j}}$, j = 1, 2.

Vibration in the bar is governed by the equations:

$$E_1 \cdot A_1 \cdot u_1''(x) - \mu_1 \cdot \omega^2 \cdot u_1(x) = 0 \quad (1a)$$

$$E_2 \cdot A_2 \cdot u_2''(x) - \mu_2 \cdot \omega^2 \cdot u_2(x) = 0 \quad (1b)$$

Where ω is the circular frequency. General solutions for these equations are:

$$u_1(x) = \sum_{n=1}^{2} A_{n,1} \cdot \exp(i \cdot k_{n,1} \cdot x) \quad (2a)$$

$$u_2(x) = \sum_{n=1}^{2} A_{n,2} \cdot \exp(i \cdot k_{n,2} \cdot x) \quad (2b)$$

The wave numbers for vibrational waves decaying respectively in the negative and positive directions of the axial coordinate can be expressed as

$$k_{1,1} = \alpha_1 \cdot \omega,\ k_{2,1} = -\alpha_1 \cdot \omega,\ k_{1,2} = \alpha_2 \cdot \omega,\ k_{2,2} = -\alpha_2 \cdot \omega \quad (3)$$

Matching conditions at the start of one cell in the periodic bar can be formulated as

$$u_1(0) = u_2(0) \quad (4a)$$

$$E_1 \cdot A_1 \cdot u_1'(0) = E_2 \cdot A_2 \cdot u_2'(0) \quad (4b)$$

Floquet theory is now used to formulate the periodicity conditions (see Figure 2):

$$u_1(L_2) = \exp(i \cdot k_B) - u_1(-L_1) \quad (5a)$$

$$u_1'(L_2) = \exp(i \cdot k_B) - u_1'(-L_1) \quad (5b)$$

These periodicity conditions are substituted into Equations (4a) and (4b), and a system of four homogeneous equations with respect to four modal amplitudes grouped in two sets, $A_{n,j}$, n = 1, 2, and j = 1, 2, is derived. This set of equations is solved for each frequency, ω, with respect to the Bloch parameter K_B, which is a standard variable in the subject. K_B provides the phase shift in a periodic wave guide at a given frequency. As derived from Floquet theory, if all roots of the characteristic equation are complex numbered, then at a given frequency no propagating waves exist in the periodic bar. Hence, wave propagation in the periodic structure is possible only when the characteristic equation has at least one purely real root K_B for a given frequency, ω. Introducing $\lambda = \exp(-iK_B)$, it means when $|\lambda| \neq 1$ for all roots of the characteristic equation, the frequencies falls into a stop band and vibration transmission within the bar is not possible. On the other hand, vibration transmission (i.e. a pass band) is characterised by $|\lambda| = 1$.

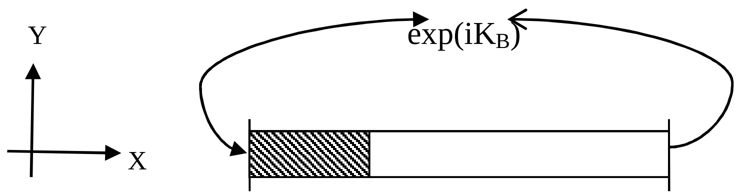

Figure 2: Formation of beam structure using Floquet approach

3 Prediction of Vibration Transmission

The insertion loss (IL) for various periodic bar structures is evaluated from the nodal displacement within the frequency range 0 to 500 Hz. Figure 3 shows the insertion-loss results for 2, 4, 6, 8 and 10 cell bar structures, respectively. As expected, the insertion loss increases with the number of cell. Thus, the higher value of the insertion loss is obtained when the bar is constructed from 10 cells. It is furthermore observed that resonance within the pass bands does not lead to infinite (or nearly-infinite) response in the finite-element model. This is a result of the impedance condition introduced in order to mimic the behaviour of a semi-infinite bar made of material 1(Plexiglas). Due to the periodicity of the structure, stop bands can be seen in all of the models from 60 to 135 Hz, 160 to 280 Hz and 300 Hz to 400 Hz.

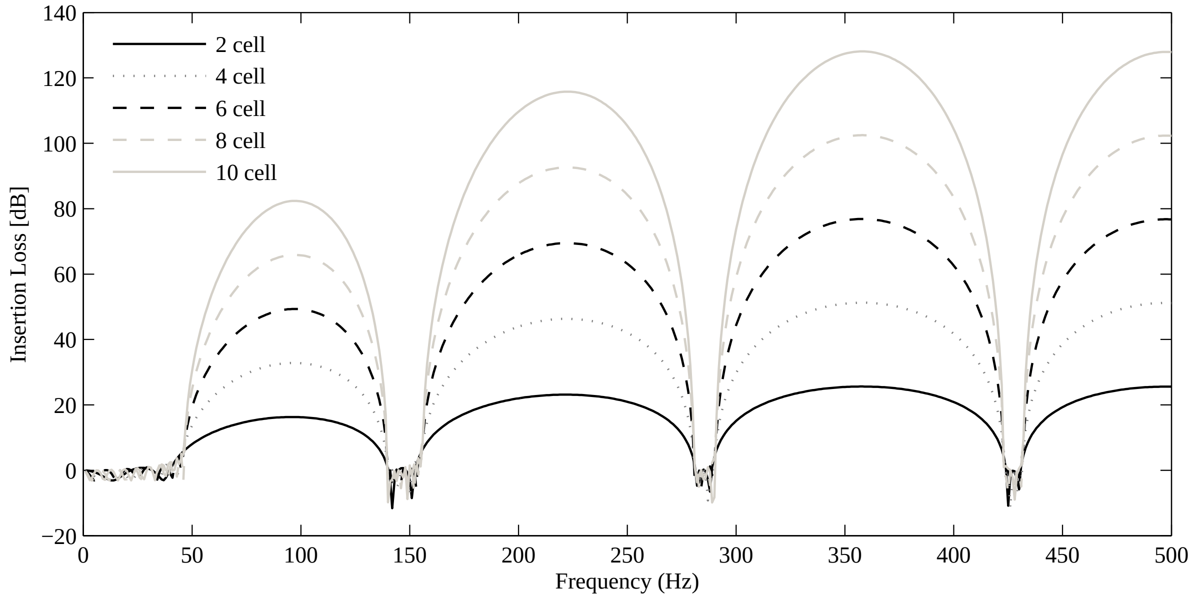

Figure 3: Insertion loss (IL) for wave propagation in the periodic semi-infinite bar structure within the 0 to 500 Hz frequency range

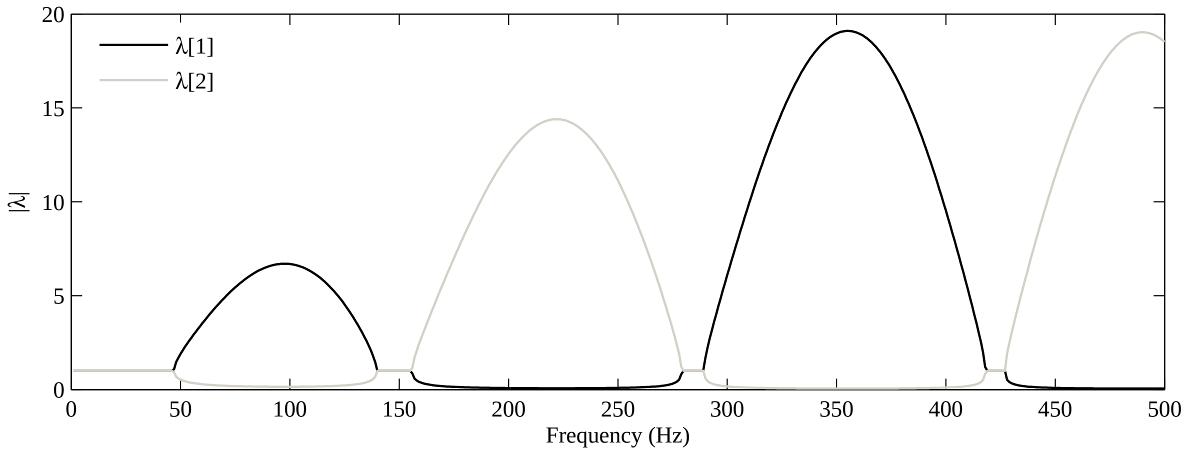

Figure 4: Bloch parameter ($\lambda = \exp(-iK_B)$) for infinite beam structure (Floquet approach)

Vibration transmission within the infinite bar structure is calculated using Floquet theory. Figure 4 shows the results in terms of the parameter $\lambda = \exp(-iK_B)$ for the infinite bar in the frequency range from 0 to 500 Hz. In Figure 4, a pass band can be easily seen in the low-frequency range, approximately up to 50 Hz, where $|\lambda_1| = 1$ and $|\lambda_2|=1$. Axial wave propagation is suppressed, i.e. stop bands are visible, in the frequency ranges from 50–140 Hz, 160–280 Hz, 300–400 Hz, where $|\lambda| \neq 1$.

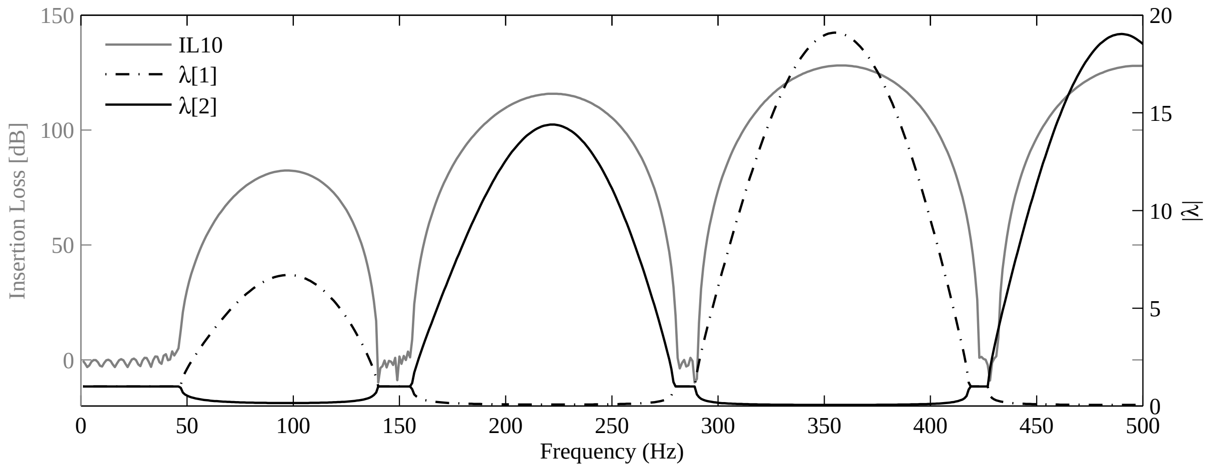

Figure 5: Insertion Loss (IL) at 10 cell periodic beam structure (FEM) and Bloch parameter (λ = exp(-iK_B)) (Floquet theory) within 0 to 500 Hz frequency range

Figure 5 shows a comparison for the evaluation of vibration transmission within the bar structure using the finite-element method (10-cell structure) and the Floquet approach. The two approaches are highly correlated in the sense that stop bands and pass bands appear in the same range of frequencies for both methods. Finally, it can be observed in Figure 3 that the stop bands are introduced already with two cells appearing in the bar structure. Thus, the frequency ranges with no wave propagation is independent of the number of repetitions of the cells along the bar—only the insertion loss changes when more cells are added to the structure.

4 Summary

Vibration transmission within a periodically connected one-dimensional semi-infinite bar structure made of Plexiglas and steel is analysed using the finite-element method (FEM) and Floquet theory. In case of the finite-element method, stiffness and mass matrices for Plexiglas and Steel elements were constructed and combined to generate a bar structure with a finite number of cells, followed by a semi-infinite bar of Plexiglas. The insertion loss (IL) for gradually increasing periodicity with 2,4,6,8 and 10 cells in the bar structure has been calculated. Stop bands with no vibration transmission are visible in certain frequency ranges. It can be seen by a comparison of the results for different numbers of cells that the insertion loss gradually increases when the number of cells increases. This proves that periodicity within the structure can reduce vibration transmission significantly.

Prediction of vibration transmission within an infinite one-dimensional infinite beam structure is also evaluated through Floquet theory, which eliminates the need to generate system matrices as in the FEM. The parameter λ = exp(-iK_B) is derived for a frequencies in the range 0 to 500 Hz, where K_B is the Bloch parameter. Pass bands are identified at certain frequency ranges when $|\lambda| = 1$ (wave propagation is visible) and stop bands identified by $|\lambda| \neq 1$ are obtained in the remaining part of the frequency range.

A comparison between the FEM and Floquet approach has been carried out. The methods have shown significant correlation in prediction of vibration transmission within the bar structure. Stop bands with no (or highly damped) wave propagation are surfaced in the frequency ranges 50–140 Hz, 160–280 Hz, and 300–400 Hz with both approaches.

Future tasks involve analyses of flexural wave propagation within a similar beam structure using FEM and Floquet theory, thus identifying the occurrence of stop bands regarding flexural waves within various frequency ranges. Results obtained by numerical and analytical methods will be compared with experimental results. The aim is to get a better understanding of sound transmission within periodically connected lightweight structures.

References

[1] R.H. Lyon, R.G. DeJong, *Theory and application of statistical energy analysis*, 2nd edition. Butterworth-Heinemann, 1996.

[2] L. Galbrun, Vibration transmission through plate/beam structures typical of lightweight buildings: Applicability and limitations of fundamental theories. *Applied Acoustics*, 71, 587–596, 2010.

[3] J. Mahn, Prediction of flanking noise transmission in lightweight building constructions: A theoretical and experimental evaluation of the application of EN12345-1, *Technical report, University of Canterbury*, Acoustics Research Group, 2007.

[4] D. Clasen, S. Langer, Finite element approach for flanking transmission in building acoustics *Building Acoustics*, 14(1), 1–14, 2007.

[5] D. Takahashi, Sound radiation from periodically connected double-plate structures. *Journal of Sound and Vibration*, 90(4), 541–557, 1983.

[6] J. Brunskog, H. Chung, Non-diffuseness of vibration fields in ribbed plates, *Journal of Acoustical Society of America*, 129(3), March-2011.

[7] B.R. Mace, D. Duhamel, M.J. Brennan, L. Hinke, Finite element prediction of wave motion in structural waveguides, *Journal of Acoustical Society of America,* 117(5), May-2005.

[8] A.S. Knudsen, On advantage of derivation of exact dynamical stiffness matrices from boundary intergral eqations (L), *Journal of Acoustical Society of America*, 128(2), August-2010.

[9] A. Safaeinili, D.E. Chementi, B.A. Auld, S.K. Datta, Floquet analysis of guided waves propagating in periodically layered composites, *Composites Engineering*, Vol 5, No 12, pp. 1471-1476, 1995.