

Energy based prediction models for building acoustics

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In order to reach robust and simplified yet accurate prediction models, energy based principles are commonly used in many fields of acoustics, especially in building acoustics. This includes simple energy flow models, the framework of statistical energy analysis (SEA) as well as more elaborated principles as, e.g., wave intensity analysis (WIA). The European standards for building acoustic predictions, the EN 12354 series, are based on energy flow and SEA principles. In the present paper, different energy based prediction models are discussed and critically reviewed. Special attention is placed on underlying basic assumptions, such as diffuse fields, high modal overlap, resonant field being dominant, etc., and the consequences of these in terms of limitations in the theory and in the practical use of the models.

1 Introduction

Instead of actually solving the wave equations, energy based principles are commonly used in many fields of acoustics, especially in building acoustics. With the use of these methods, robust and simplified yet accurate prediction models can be developed. One important reason for the wide use of energy methods in building acoustics is that the predictions have to cover the broad audible frequency range. Another important aspect is that in building acoustics, measurements are made in 1/3 octave bands, and as the energy based methods in general gives broad band estimates, this suits fine. The considered methods includes simple energy flow models, the framework of statistical energy analysis (SEA) as well as more elaborated principles as the wave intensity analysis (WIA). An energy based method simply means that basic variables being solved for is related to energy (or power or intensity), and not the field variables with phase information such as sound pressure or vibration velocity. The use of root-mean-square sound pressure and uncorrelated addition are thus examples of energy based calculations. In SEA [1,2] the studied structure is divided into a number of subsystems in each of which the vibrational energy is assumed to be evenly distributed. The European standards for building acoustic predictions, the EN 12354 series [3], are based on energy flow and SEA principles [4]. The energy based predictions are used in all kinds of predictions, but perhaps most importantly in the prediction of flanking transmission. Flanking transmission means transmission of sound through the flanking partitions, as opposed to direct sound transmission through the building element. The EN 12354 series provides prediction models for flanking transmission, and not for direct path predictions, using measured data as input.

Lightweight buildings are probably the future in building technology. Multi-story buildings with lightweight framing systems are of major interest because the production costs are usually lower, especially in prefabricated production; low weight implies simpler and cheaper building foundations and lower transportation costs; and wooden frames are an environmentally friendly material. The project of *Välle Broar* is an example [5], being an entire city area where wooden buildings are built with new techniques. High-rise lightweight wooden frame buildings have also been developed in Denmark and several other countries [6]. Other important examples of lightweight building systems are steel frame structures and lightweight building elements of materials like plaster-boards, chip-boards and mineral wool. However, lightweight building structures are known to have poor sound insulation because of sound transmitted via edges and flanking paths. Flanking transmission is very difficult to measure in a laboratory, unlike the direct transmission path. Energy based prediction models for flanking transmission have been available for grossly homogeneous structures such as concrete for a long time [1,7]. However, these models are not applicable to lightweight stiffened constructions, as

will be explained in this paper. When no prediction model is available, the only alternatives are to use traditional building solutions, or to build expensive test houses; none of these alternatives are good.

2 Basic principles and statistical energy analysis (SEA)

2.1 Energy flow and uncorrelated addition

Energy flow methods in this paper refer to all energy based methods that are not based on the more elaborated theories as SEA. Often, but not necessarily, is an energy balance used, just as in SEA. The basic variables are related to energy. The use of uncorrelated addition of root-mean-square quantities in the derivation of a prediction model is often used. The calculation standard EN 12354 was originally derived as an energy flow prediction model [7,8], but it was later shown that it can be derived using first order SEA [4]. One of the key points of the Gerretsen and EN 12354 models are the averaging between the transmission in one direction and the reciprocal transmission in the opposite direction, combined with the use of measured data for direct sound insulation. In this way knowledge of the radiation efficiencies is avoided, which simplifies the model.

As an example of the energy flow methods, the models for the structure borne transmission of double leaf partitions (wall, windows, floors) will be used. The model for double constructions with sound bridges by Sharp [9] is a simplified yet good example of such a model. Similar are the models by Vigran [10] and Davy [11], both being slightly more elaborated than Sharps model. The sound bridges are typically wooden or metal, providing a direct mechanical connection between the two plates. The basic assumption is that the power radiated due to the action of the bridge, P_b , can be added to the power radiated by the ideal panel with only airborne transmission in the cavity, P_p . This is a good assumption in most cases, as the two vibration fields consist of different types of wavenumbers (forced and resonant waves). However, at the coincidence the wavenumbers of the forced and resonant vibration fields are by definition equal, and power addition is thus more problematic. Sharp proceeds by considering the transmission via one sound bridge (a stud), being assumed infinitely stiff and weightless. In the next step is it assumed that also the sound power radiated from the individual sound bridges are uncorrelated and can be simply added. If assuming the studs to be randomly distributed, this is a good assumption. However, real building structures typically have equal distances between elements, which form a periodic system, see more below in section 3.2. Periodic structures have altering frequency/angle bands with wave propagation, pass bands, and altering bands with attenuation, stop bands. This is not reflected in Sharps assumption of uncorrelated addition of the sound power radiated from the individual sound bridges. Sharps model (and similar theories) are however quite accurate in their predictions. This is probably due to the ‘smoothing’ due to the random process of diffuse airborne excitation. More problematic with the models by Sharp, Davy and Vigran is the behaviour in the low frequency region around the mass-spring-mass resonance. Sharp’s model includes the mass-spring-mass resonance frequency (as calculated by the mass per unit area of the leafs and the bulk stiffness of the air cavity) as a transition frequency where a low frequency model, using the total mass law, is shifted to the double construction model. But the actual resonance dip around this frequency is not modelled, and the effect of the mechanical connection is not included when predicting the mass-spring-mass resonance frequency. This effect has been discussed by Bradley and Brita [12]. In any case, in Hongisto’s investigation of the accuracy of the available simplified prediction models for double leaf partitions, the models by Sharp and Davy turned out to be the best choices in their categories [13], with an average prediction error of around 5 dB for R_w . These results are actually better than the SEA models examined (being the model by Price and Crocker [14], discussed later on), which might be surprising as SEA models in some sense are more elaborate. On the other hand, the energy flow methods are much flexible, making room for empirical observations to be included in the model. This is one important reason that the models by Sharp, Davy and Vigran works so well. The empirical observations being included in the models are the division of different frequency ranges with different results, which are constructed to overlap.

2.2 Statistical energy analysis (SEA)

The method of Statistical Energy Analysis (SEA) [5,6] was originally developed by Lyon and Maidanik [15] and applied for sound insulation by, e.g., Crocker and Price [14] and Vér and Holmer [16]. The method has been useful for investigation of sound propagation through complicated solid structures, like trains, ships and buildings. SEA is a modelling framework based on theory derived for weakly coupled oscillators in which the steady-state power flow between the coupled system is determined. The studied structure or sound field is divided into a number of subsystems, of which in each the acoustical energy is assumed to be evenly distributed. The weakly coupled oscillators are identified as the resonant modes within the subsystems. The subsystems are typically associated with the physical sub-

elements of which the structure is composed, that is, by plates, beams, etc, as well as the acoustic fields of rooms and air cavities. The subsystems should contain a group of modes with similar properties and energies.

The total mechanical energy in a vibration system is the sum of the kinetic and the potential energy. The kinetic and the potential energy are on average equal. It is common to describe mechanical vibrations by the vibration velocity of the surface, and thereby the kinetic part of the energy. The total energy E in a vibrating plate can be expressed as twice the kinetic energy, $E = m''S\langle\tilde{v}^2\rangle$, where m'' is mass per unit area, S is the area of the plate, and $\langle\tilde{v}^2\rangle$ is the mean square of the velocity averaged over the area of the plate. For the sound field in rooms and air cavities the energy is instead described by the sound pressure, and thereby the potential part of the energy. Energy losses are described by the loss factor,

$$\eta = \frac{P_d}{\omega E} \approx \frac{2.2}{fT_{60}}, \quad (1)$$

defined as the dissipated energy in a time period of vibration divided by 2π and the total mechanical energy. P_d is the dissipated power, $\omega = 2\pi f$ is the angular frequency, f is the frequency (the centre frequency of the frequency band under consideration) and T_{60} is the reverberation time.

If energy can be transferred from the normal modes in one resonant system to the normal modes in another system, the two systems are coupled. This means that two conditions are to be fulfilled: the systems must have a physical connection, and the systems must have normal modes with natural frequencies sufficiently close to each other, in order that the energy can be exchanged. The coupling between subsystems is described by a matrix of coupling loss factors (CLFs). The definition is analogue to the definition of the loss factor; the CLF η_{ij} from system i to system j is

$$\eta_{ij} = \frac{P'_{ij}}{\omega E_i}, \quad (2)$$

where P'_{ij} is the power transferred from system i to system j and E_i is the total energy in system i . The CLF between different type of systems can then be related to transmission coefficients τ_{ij} or radiation efficiencies σ_{ij} , which are known from other theories, or can be measured. When the energies, injected powers, loss factors and CLF for the different subsystems are identified, a power balance established for each subsystem, and the unknown subsystem energies can be solved for.

The consideration of the energy balance between the two systems should be made on the basis of the normal modes that carry the energy. It is the energy per mode in each system that is important for the energy balance, as it is the modes that are the degree of freedom of the system. In system i the average energy per mode $E_{m,i}$ is

$$E_{m,i} = \frac{E_i}{\Delta N_i} = \frac{E_i}{\Delta f} \frac{1}{n_i(f)}, \quad n_i(f) = \frac{\Delta N_i}{\Delta f}, \quad (3)$$

where ΔN_i is the number of modes in the frequency band Δf , and $n_i(f)$ is the modal density. The modal density represents the number of resonant modes of the subsystem within a frequency band. The net energy transfer per unit time from system i to system j is

$$P_{ij} = \omega \Delta N_i \eta_{ij} E_{m,i} - \omega \Delta N_j \eta_{ji} E_{m,j}. \quad (4)$$

A fundamental idea within SEA is that each normal mode in a resonant system can be considered a degree of freedom, and it is a general physical principle that energy transfer between coupled systems will always be in the direction from the system with higher energy per degree of freedom to the system with lower energy per degree of freedom, c.f. with statistical mechanics. This implies that for modal energy balance, $E_{m,i} = E_{m,j}$, the net power flow P_{ij} is zero, and therefore

$$\eta_{ij} = \frac{\Delta N_i}{\Delta N_j} \eta_{ji} = \frac{n_i(f)}{n_j(f)} \eta_{ji}, \quad (5)$$

which is the reciprocity relation for the coupling loss factors, being a central part of SEA.

As examples of SEA the models for airborne sound insulation of double leaf partitions will be used again, including both airborne transmission in the cavity and mechanical transmission via the studs. Two well known models are the model by Price and Crocker [14] from 1970 (without studs), and the model by Craik and Smith [17,18] from 2000 (with and without studs). The procedure to develop a SEA model is to first identify the SEA subsystems, and to establish how these are coupled. Craik and Smith uses different models, with different numbers of subsystems, in different frequency ranges and depending on the fact if point or line connections can be assumed for the coupling between plates and studs.

In the low frequency range is a 3 subsystem model used (room, wall and room), including a forced field direct coupling between the rooms. In the high frequency range when no mechanical coupling is included is 5 subsystems used (room, wall, cavity, wall and room), again including a forced field direct coupling between the rooms and cavities. When structural line coupling via the studs are present, again 5 subsystems are used (room, wall, cavity, wall and room), but now also including a direct coupling between the walls. It is a problematic fact that different models are used in different frequency ranges, and that they do not overlap at the crossover frequencies (as the mass-spring-mass frequency). Moreover, discussed above, the SEA model by Price and Crocker [14] gives worse result than the energy flow models in Hongisto's validation paper [13]. Price and Crocker's model do not include the 3 subsystem low frequency model that Craik and Smith uses, and do have a poor precision in this frequency range. Effects like the mass-spring-mass frequency is difficult to model in SEA.

3 Energy methods applied to modern building structures

Even if SEA is a powerful tool, there are some general underlying assumptions that put restrictions and drawbacks on the methodology, especially when applied to modern building structures. Fahy [19] has given a critical overview of SEA. Some of the underlying assumptions are: that only resonant fields are of importance; that the modal density should be high and smooth; that there should be a more or less diffuse field; and that the coupling between different subsystems should be weak. Some of these assumptions can be violated if one is careful, but it is important to be aware of these underlying facts. One consequence of the underlying assumptions is that structures that are periodic in nature – as in many lightweight and wooden building structures – are not well suited for SEA.

3.1 Periodicity, modal density and spatial attenuation

Plates reinforced by ribs with equal distance in between are common elements in lightweight building structures, as well as in other engineering structures (vehicles, ships). In its present state of development, SEA has severe difficulties in handling such periodic structures. The three basic SEA assumptions that are problematic for such structures are: that only resonant fields are of importance; that the modal density should be high and smooth; and that there should be a more or less diffuse field. Fahy [19] discusses some of these problems. The latter, the problems due to lack of diffuseness, is discussed in section 3.2, and the former is discussed in section 3.3.

The modal density for common building elements like plates, beams and air cavities are on average smooth functions of the frequency. However, for periodic structures this will not be the case. Periodicity in structures allows waves to propagate freely in certain frequency bands, the pass bands, and result in attenuation in other frequency bands, the stop bands, see, e.g., Mead [20]. For a finite one-dimensional (1D) periodic structure the result is that the modes group together in pass bands, and no modes are present in the stop bands. The modal density is then not a smooth function of the frequency. In 2D, as in a rib-reinforced plate, the occurrence of pass bands and stop bands depends on the propagation angles, and the modes group together in pass bands. However, the uneven modal densities can be compensated for in certain cases. Keane and Price [21] modelled the uneven modal density in a 1D systems, and Tso and Hansen [22] included a 2D rib reinforced plate in a SEA system, using the modal density of such a structure, as developed by Langley [23]. In Refs. [24, 25] the modal density of ribbed plate are investigated, using a modal model of a ribbed plate. Some suggestions are given regarding how to proceed towards a SEA based prediction model for ribbed plates, using a subdivision of the modes.

Associated with the periodicity is the high and directional spatial attenuation in these structures. There will be a highly directional spatial attenuation in the considered structures, with high attenuation orthogonal to the ribs. Examples can be found in the measurements presented by Sjökvist [26, 27]. The measurements were conducted on a lightweight wooden floor construction, with a junction between a source floor and a receiver floor. The attenuation rate away from a tapping machine at the source floor in the direction orthogonal to beams were 4-10 dB/m, and the same attenuation rate at the receiver floor were 1.5-6 dB/m. The attenuation rate in the direction parallel to beams were much lower, and sometimes actually increasing, -2 to +3 dB/m. The attenuation is partly due to the attenuation in the stop bands, but also due to the imperfect periodicity, causing energy localisation near the source (Anderson localisation) [28, 29]. This implies that there is no simple relation between the energy density and the transmitted power, making it difficult to find the CLFs.

3.2 Non-Diffuseness

The diffuseness assumption is commonly used within SEA, not as part of the energy conservation equations but for determining the coupling loss factors. However, in rib-stiffened plate structures, the overall vibration field is not diffuse due to the ribs [30]. This limits the use of standard SEA on such structures. Numerical simulations have been used to show that the structural intensity in the rib-reinforced plates varies significantly with respect to the angle of the wave propagation. The angular distribution of the structural intensity is consistent with the behaviour of periodical structures in terms of pass and stop bands; this is also seen in one-third octave band averaged results. However, for uniform plates without rib stiffeners, the field is diffuse in an ensemble average sense, and these structures should thus be better suited for SEA. When the assumption on diffuseness is not fulfilled the vibration field will have a complicated relationship to the structural transmission loss and the CLFs. The lack of diffuseness is most problematic for flanking transmission where wave propagation along the structure is important. For the direct transmission through the structure a diffuse field assumption of the wave propagation is never needed.

Also sequences of similar structures – without any periodicity involved – are likely to cause problems within SEA. Langley and Bercin [31] have given an example with a structure consisting of a chain of 15 plates. Conventional SEA and wave intensity analysis (WIA), see section 4.2, were compared with measurements. WIA predicts the measured results well, whereas SEA underestimates the results. The discrepancy increased as the distance between the source and the measurement location grew. At the last plate, the error was about 30 dB. Langley and Bercin explained this with the filtering effect of the structural junctions not being considered by SEA. In other words, the waves close to normal to the junctions usually have a higher transmission coefficient in comparison to other wave directions. Therefore, the angular-averaged transmission coefficient used in SEA is lower than what it should be. Considering the entire plate, this filtering can be seen as lack of diffuseness.

3.3 Forced and resonant fields, and sound radiation

One of the basic assumptions of SEA is that only resonant fields are of importance. An example of a resonant field is the free bending wave in a plate. However, for airborne sound insulation is also the forced field – or in SEA sense the ‘non-resonant field’ – of large importance, being caused by the exciting pressure field in the surrounding air. In their SEA model for airborne sound insulation through a simple homogeneous structure, Vér and Holmer [16] solved this problem by adding a direct coupling from the receiver room to the source room, describing the non-resonant transmission. However, the distinction in separate resonant and non-resonant fields is not unproblematic. There is no generally agreed definition of the two fields. One definition is based on the wavenumbers: the forced field have forced wavenumbers and the resonant field have free wavenumbers. Another definition is based on SEA: the resonant field is what is included in the SEA description, and the non-resonant field is what is left (which is a self referring definition, which is problematic) or simply the mass law. Lyon and DeJong [2] use a description of transmission through a direct coupling that at low frequencies is the mass law, and a resonant transmission that ‘is the indirect coupling through the resonant modes of the plate’. The actual physical fields consists of waves or modes (depending on the perspective). But these can only approximately be divided in two bins corresponding to forced/non-resonant or free/resonant behaviour. Using a modal description of a plate excited by an acoustic air field, also the forced part is described by the modes. If a wave number definition is used, the division is most problematic at and above the critical frequency – the forced and resonant fields are by definition equal at coincidence.

In the prediction standard EN 12354 the forced and resonant fields are even more problematic, due to the fact that it is assumed that only the resonant part of the transmission is contributing. This assumption was used together with the reciprocal transmission path in order to avoid the use of radiation efficiencies. Thus EN 12354 in its present form is not suited for predictions below the critical frequency, which for lightweight building structures correspond to most of the frequency bands of interest. Different solutions to this problem have been suggested, e.g., see Refs. [32, 33], but no really convincing solution is yet found.

Another problem associated with the forced and resonant fields in a lightweight building structure is their contribution to radiation. The sound radiation property of a structure with an irregular vibration field is very complicated, and for rib stiffened plates, the expressions for the radiation efficiency is not as well developed as for uniform plates. Vigran [10] has recently studied the radiation efficiency of ribbed plates, but without accounting for the spatially periodic phase variation and associated attenuation.

3.4 Double plate layers and coupling at the joint

Lightweight structures are often double leaf constructions. At low frequencies the two plates will act as one unit, but they are loosely coupled at higher frequencies. The problem is how to treat the structures in between these extreme ends. This is a major problem in the SEA models available for the direct airborne transmission through lightweight building structures, e.g. the models by Price and Crocker [15] and Craik and Smith [17, 18], as discussed in section 2.2. The behavior around the mass-spring-mass resonance cannot be predicted with an overall approach assuming separate transmission paths for the airborne and the structure borne transmission, being added in energy sense. Instead, a full coupled theory has to be used, as in Lin and Garrelick [34], Takahashi [35] or Brunskog [36]. But these are not energy based models.

The periodic nature of the wave media will affect the amount of power transmitted through the junction (and thus the CLFs), as shown by Brunskog [37], studying free waves travelling in a periodic wave-medium. The free waves meet a discontinuity (or junction), and the power transmitted through the discontinuity is then studied. From the numerical results it can be concluded that the periodic nature of the surrounding media do affect the amount of power being transmitted through the junction. In the case of a periodic structure, with a simple support at the discontinuity, high transmission occur at some frequencies and total attenuation at others, whereas the corresponding uniform case has a constant transmission of $\tau = 1/2$. These effects were not taken into account in the SEA model including a 2D rib reinforced plate by Tso and Hansen [22], nor in the available SEA models for flanking transmission [38, 39].

4 Alternative approaches

4.1 Wave intensity analysis (WIA)

The wave intensity analysis (WIA) was developed by Langley [40]. WIA incorporates non-diffuseness into SEA using a Fourier expansion of the wave intensity, taking the propagation in different directions into account. The intensity 'rays' are studied primarily, and the phase information is lost. WIA is an elaborate and yet simplified prediction model. It can be noted that the theory of WIA gives interesting results regarding "angular filtering" of the intensity field due to rib joists.

5 Discussion

An obvious advantage of the energy based methods, including SEA, is their simplicity and low computational effort. The amount of information used to account for the physics involved is minimal, each building element being described by a number (for each frequency); the mean of the sound energy being obtained for each part or component of the structure separately. In Hongisto's [13] investigation, energy flow models were shown to be better at predicting the sound insulation of double walls, as compared to simple wave based models. However, the fact that a minimal amount of information is utilized represents both an advantage and a disadvantage. These approaches are particularly appropriate for dealing with homogeneous and clearly distinguishable building elements, such as in traditional building construction systems in which the elements are heavy and homogeneous. Such approaches are not likely to be successful, however, if one's interest is in discovering new types of solutions, since the lack of the information needed makes it impossible to describe the physics of the situation adequately. Moreover, for heavy and homogeneous traditional building structures, the underlying assumptions are well met. This will however not be the case for modern lightweight building elements and constructions. How to incorporate these structures in the framework of SEA might be the most urgent and important research field regarding predictions in building and structural acoustics. As the prediction standard EN 12354 [3] is closely related to SEA, it shares many of the basic assumptions and limitations with SEA.

As compared to SEA, the simpler energy flow methods seem to be better suited for constructing prediction models for the direct transmission of building structures. This is due to the fact that these methods are more flexible, and can therefore more easily incorporate empirical observations.

6 Conclusions

Lightweight building constructions have several problems with violation of the underlying assumptions of SEA. Effects like the mass-spring-mass frequency is difficult to model in SEA. The simple flexible energy flow methods seems better suited for predicting sound insulation than a strict use of SEA.

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